

# ImQMD model and its applications in HICs

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# Outline

1. Brief Introduction to the ImQMD model
2. Applications to heavy ion reactions for constraining the density dependence of symmetry energy
3. Summary

# 1, Introduction to the ImQMD model

## ∅ What is the meaning of ImQMD?

(Improved Quantum Molecular Dynamics Model)

## ∅ Why did we make these improvements?

Study the low and intermediate energy heavy ion reactions with QMD type model

## ∅ What did we do in ImQMD?

Two versions:

- ImQMD (for lower energy reactions, such as fusion, the interaction parameters are adjusted specially)

Wang, Li, Wu, PRC**65**,064608(2002),  
C**69**,024604(2003), C**69**,034608(2004)

- ImQMD05

Zhang, et al., PL **B664** (08) 145, PR  
C**71** (05) 024604, PR C**74** (06) 014602



## ImQMD05

∅ the mean fields acting on nucleon wavepackets are derived from Skyrme potential energy density functional

$$\dot{\mathbf{r}}_i = \frac{\partial H}{\partial \mathbf{p}_i}, \quad \dot{\mathbf{p}}_i = -\frac{\partial H}{\partial \mathbf{r}_i}, \quad H = T + U_{\text{loc}} + U_{\text{Coul}},$$

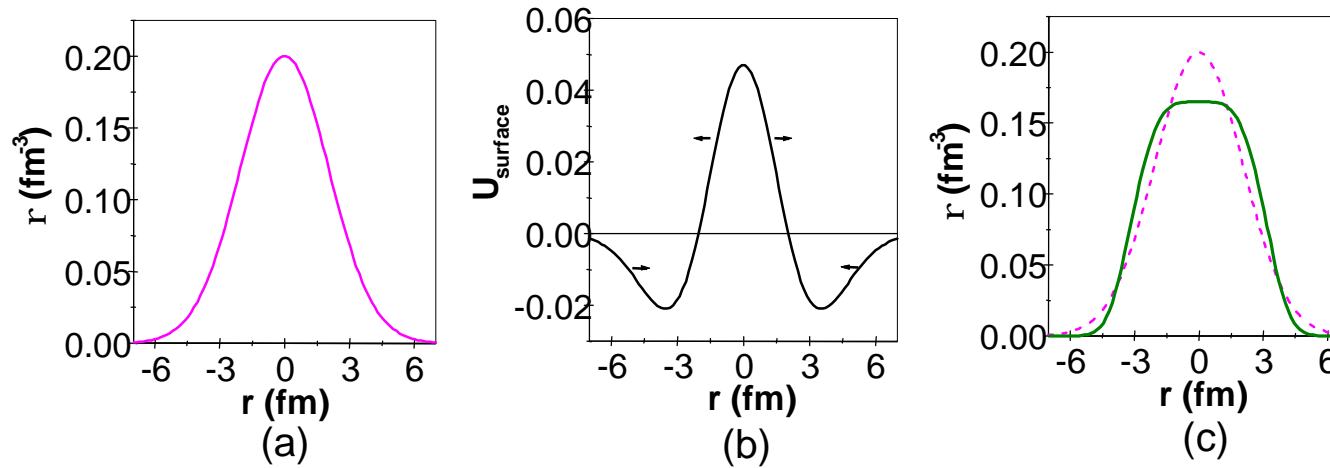
$$U_{\text{loc}} = \int V_{\text{loc}}[\rho(\mathbf{r})] d\mathbf{r}.$$

potential energy density functional:

$$V_{\text{loc}} = \frac{\alpha}{2} \frac{\rho^2}{\rho_0} + \frac{\beta}{\eta + 1} \frac{\rho^{\eta+1}}{\rho_0^\eta} + \frac{g_{\text{sur}}}{2\rho_0} (\nabla\rho)^2 + \frac{g_{\text{sur,iso}}}{\rho_0} [\nabla(\rho_n - \rho_p)]^2$$
$$\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$$
$$+ (A\rho^2 + B\rho^{\eta+1} + C\rho^{8/3})\delta^2 + g_{\rho\tau} \frac{\rho^{8/3}}{\rho_0^{5/3}}$$

Parameters in  $V_{\text{loc}}$  are obtained from standard Skyrme interactions parameters

## U Surface and surface symmetry energy term



## Ø Pauli blocking with uncertainty relationship

$$P_{\text{block}} = 1 - (1 - P_i)(1 - P_j).$$

$$P_i = 0.5 \sum_{k, k \neq i} \frac{1}{(\mathbf{ph})^3} \exp\left[-\frac{(r_i - r_k)^2}{2s_r^2}\right] \exp\left[-\frac{(p_i - p_k)^2}{2s_p^2}\right]$$

$$\frac{4\pi}{3} r_{ij}^3 \cdot \frac{4\pi}{3} p_{ij}^3 \geq \frac{h^3}{8}$$

(enhance the Pauli blocking at lower energy and efficiency in calculations)

In addition ,

- Phase space constraint  $\bar{f}_i \leq 1$  for considering Pauli principle

The occupation number in cubic of  $h^3$  should be smaller than 1 (Pauli principle)

- Isospin dependent nucleon-nucleon cross sections are adopted, the medium corrections are

$$S_{np}^{med} = (1 - h r / r_0) S_{np}^{free}$$

$$S_{nn,pp}^{med} = (1 - h r / r_0) S_{nn,pp}^{free}$$

$$S_{np,nn(pp)}^{free} \quad \text{Cugnon, et al., Nucl.Instr.Meth.Phys. B111, 215(1996)}$$

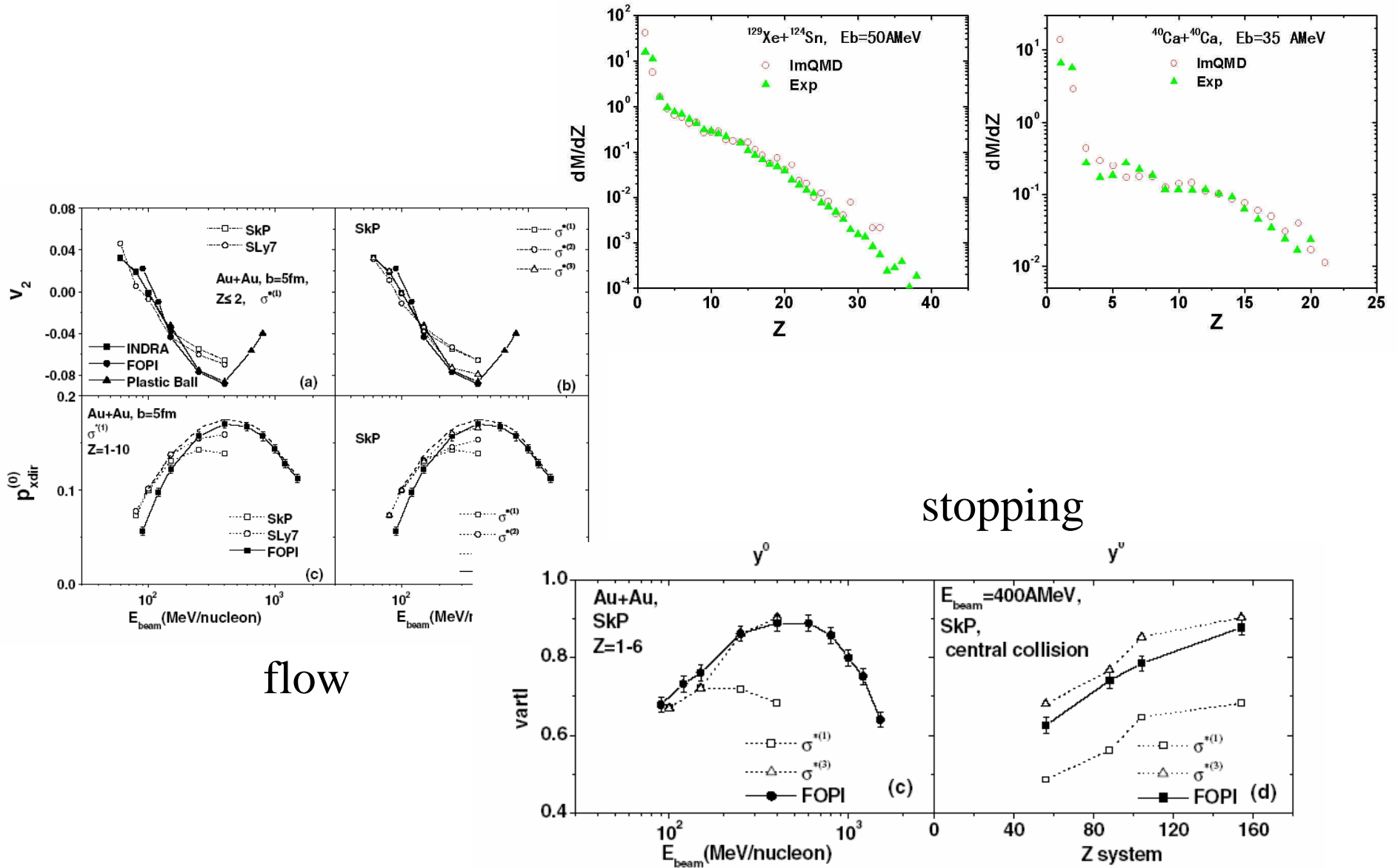
- Momentum dependence interaction (??) **Aichelin, et al., PRL58,1926(1987)**

$$u_{md} = \frac{1}{2r_0} \sum_{N_1, N_2=n,p} \frac{1}{16p^6} \int d^3 p_1 d^3 p_2 f_{N_1}(\mathbf{p}_1) f_{N_2}(\mathbf{p}_2) 1.57 \left[ \ln(1 + 5 \times 10^{-4} (\Delta p)^2) \right]^2$$

- Clusters are constructed by means of the coalescence model widely used in QMD calculations,  $dr_{ij} \leq 3.5 \text{ fm}$ ,  $dp_{ij} \leq 250 \text{ MeV}/c$

# Some results from ImQMD05:

Zhang, Li, PRC71(2005)24606



Data: Residorf, PRL 92(2004)232301

## II. Applications to HI reactions for constraining the density dependence of symmetry energy $E_{\text{sym}}(\rho)$

∅ How to constrain the EOS on earth?

By comparing the transport model simulation results with the HICs experimental observables.

But

There are theoretical uncertainty on transport model simulations results from different model.

*Two type model: BUU type, QMD type*



## *Ø What are the differences between transport models?*

### **BUU models:**

*Semiclassical solution of one-body distribution function.*

#### **Pros**

*Derivable, approximations better understood.*

#### **Cons**

*Mean field  $\Rightarrow$  no fluctuations  
BUU does not predict cluster formation*

### **QMD:**

*Molecular dynamics with Pauli blocking.*

#### **Pros**

*Predicts cluster production*

#### **Cons**

*Cluster properties (masses, level densities) approximate  
Need sequential decay codes to de-excite the hot fragments*

At high incident energies, cluster production is weak  
 $\Rightarrow$  the two type models yield the same results.  
Clusters are important in low energy collisions.

*Further differences between different transport models:*

- 1. the mean field,  $m^*$ , MDI,*
- 2. in-medium NN cross sections,*
- 3. Pauli blocking*

*Uncertainties*

*Semi-classical Approximations  
needed to make computation feasible.*

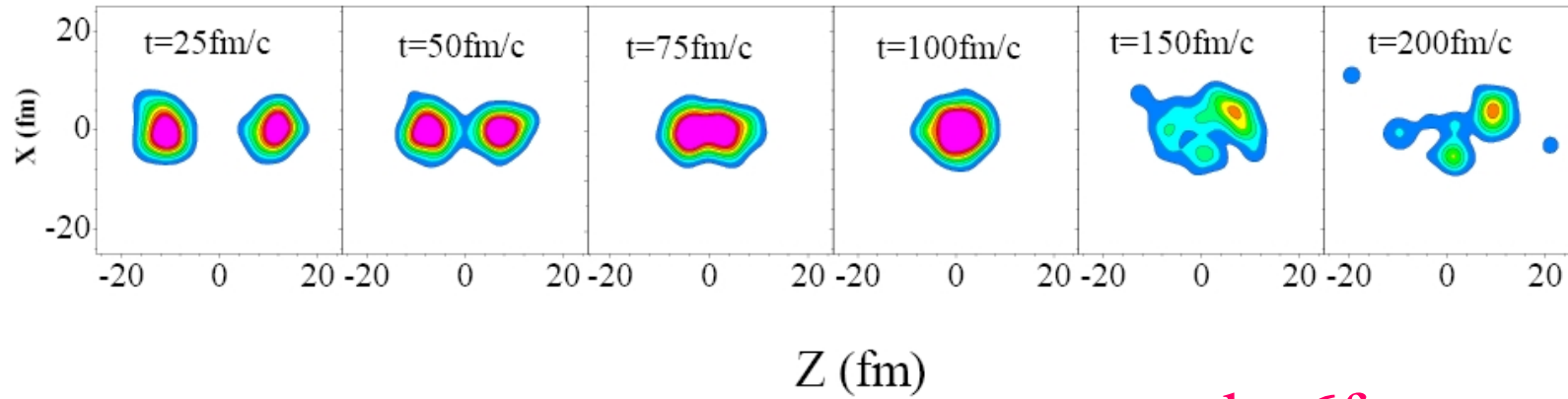
*Width of wavepacket, test particles*

*What did we get from ImQMD model simulations ?  
Compare with other models, what can we learn?*

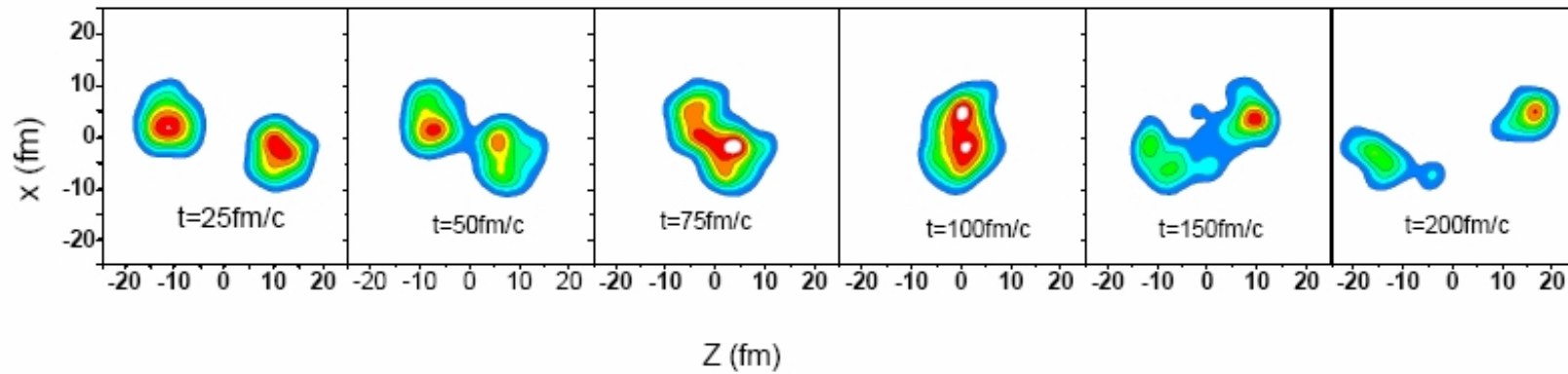
# 1), reaction mechanism

$E/A=50\text{MeV}, \text{Sn}+\text{Sn}$

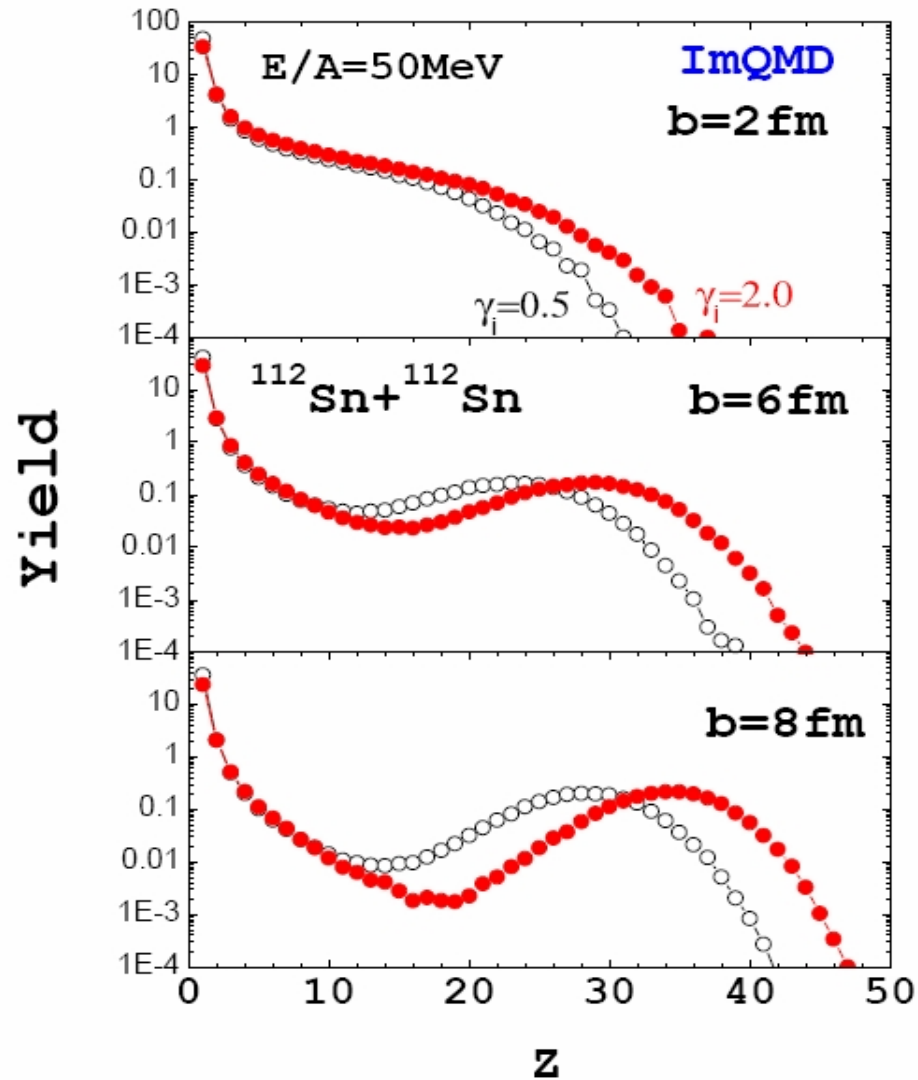
$b=0\text{fm}$



$b=6\text{fm}$



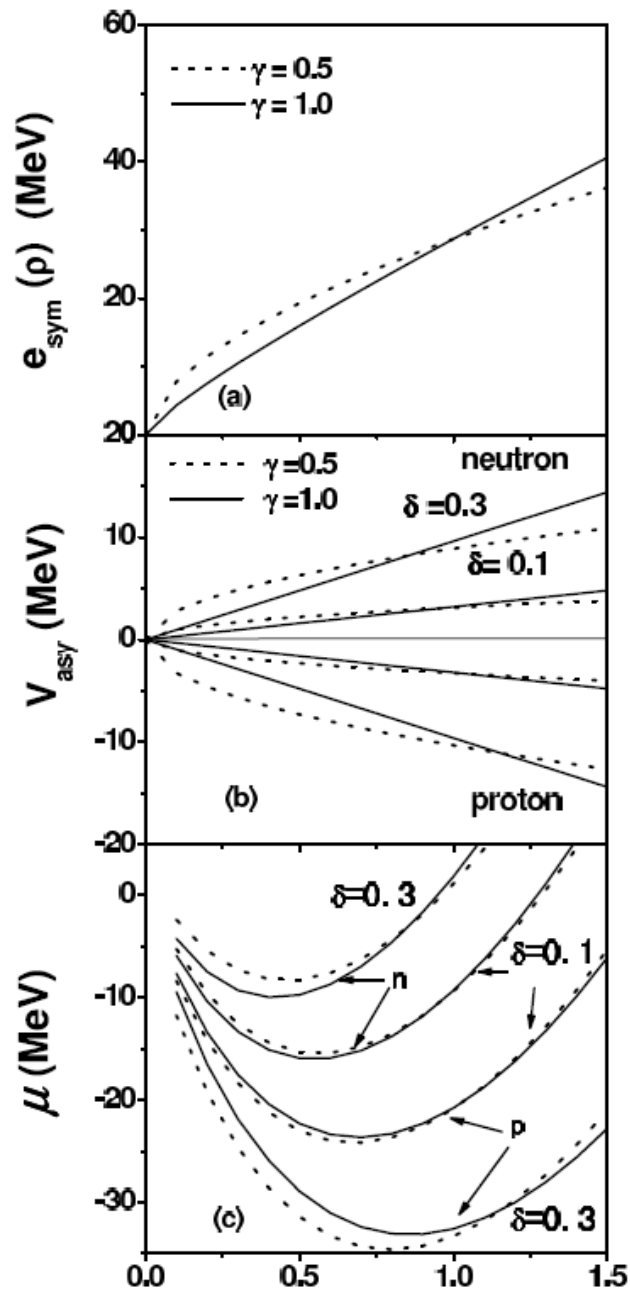
# charge distributions



Reaction mechanism of fragments productions depend on the impact parameters

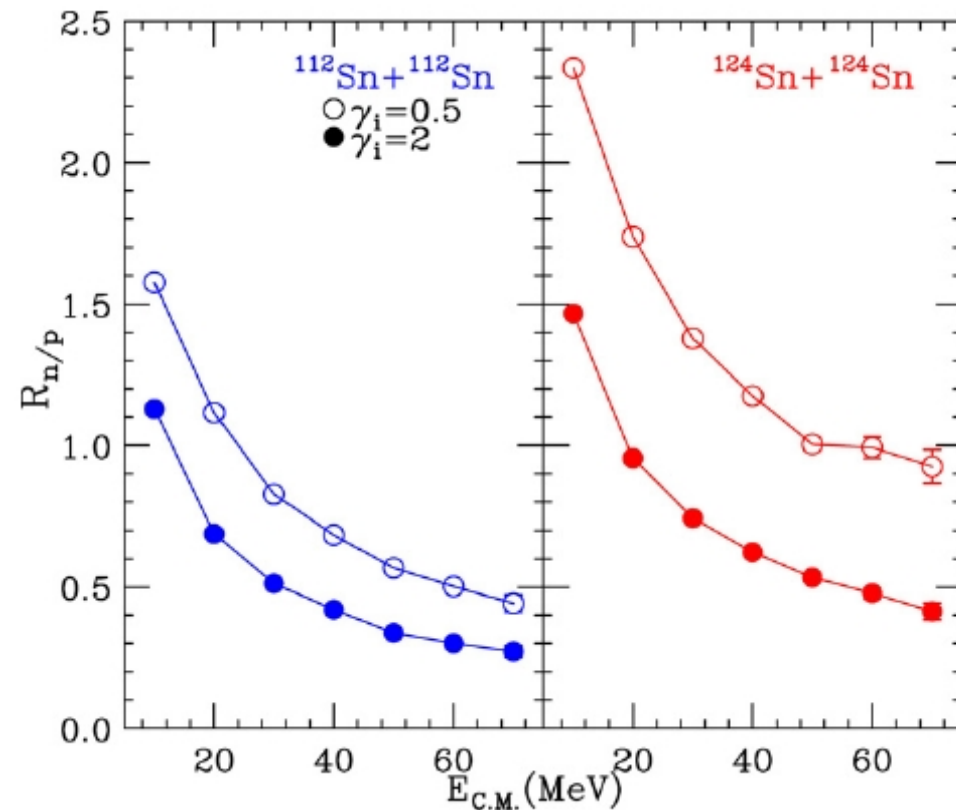
Charge distributions depend on the stiffness of EoS

## 2), n/p ratios for emitted nucleons



- $n$  and  $p$  potentials have opposite sign.
- $n$  &  $p$  energy spectra depend on the symmetry energy  $\tilde{a}$  softer density dependence emits more neutrons at low density.

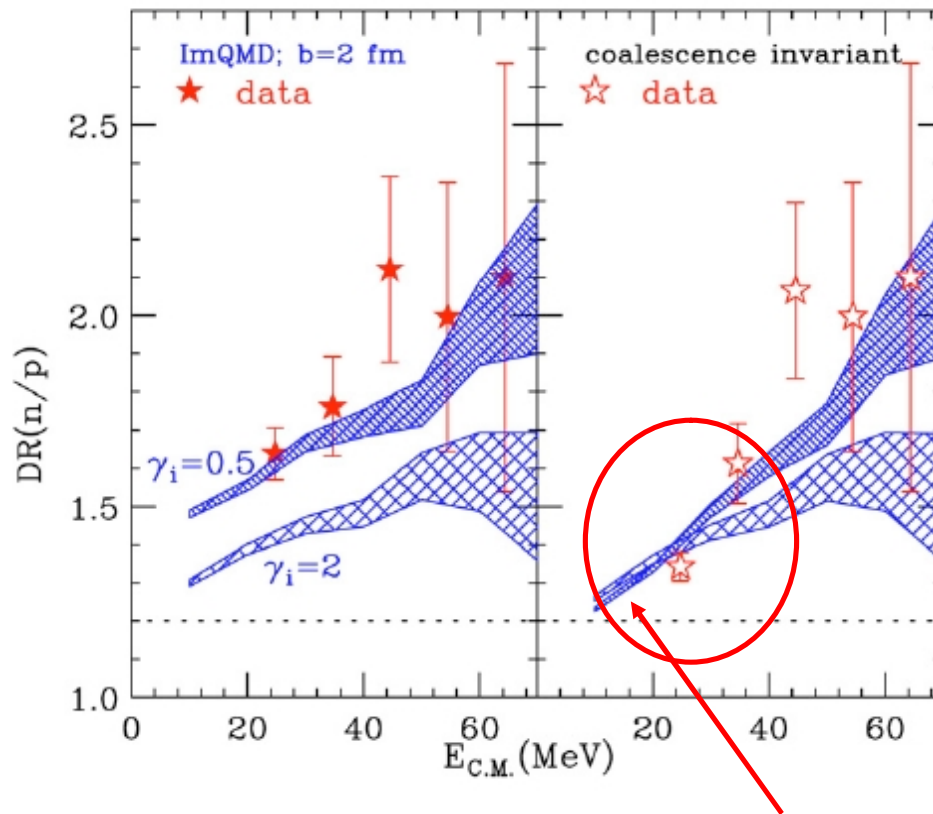
$$E_{\text{sym}}(r) = 12.5(r/r_0)^{2/3} + 17.6(r/r_0)^{g_i}$$



# n/p Double Ratios (central collisions)

Double Ratio  $\Rightarrow \frac{{}^{124}\text{Sn}+{}^{124}\text{Sn}; Y(n)/Y(p)}{{}^{112}\text{Sn}+{}^{112}\text{Sn}; Y(n)/Y(p)} \Rightarrow$  minimize systematic errors

Zhang, et al., PL **B664** (08) 145,

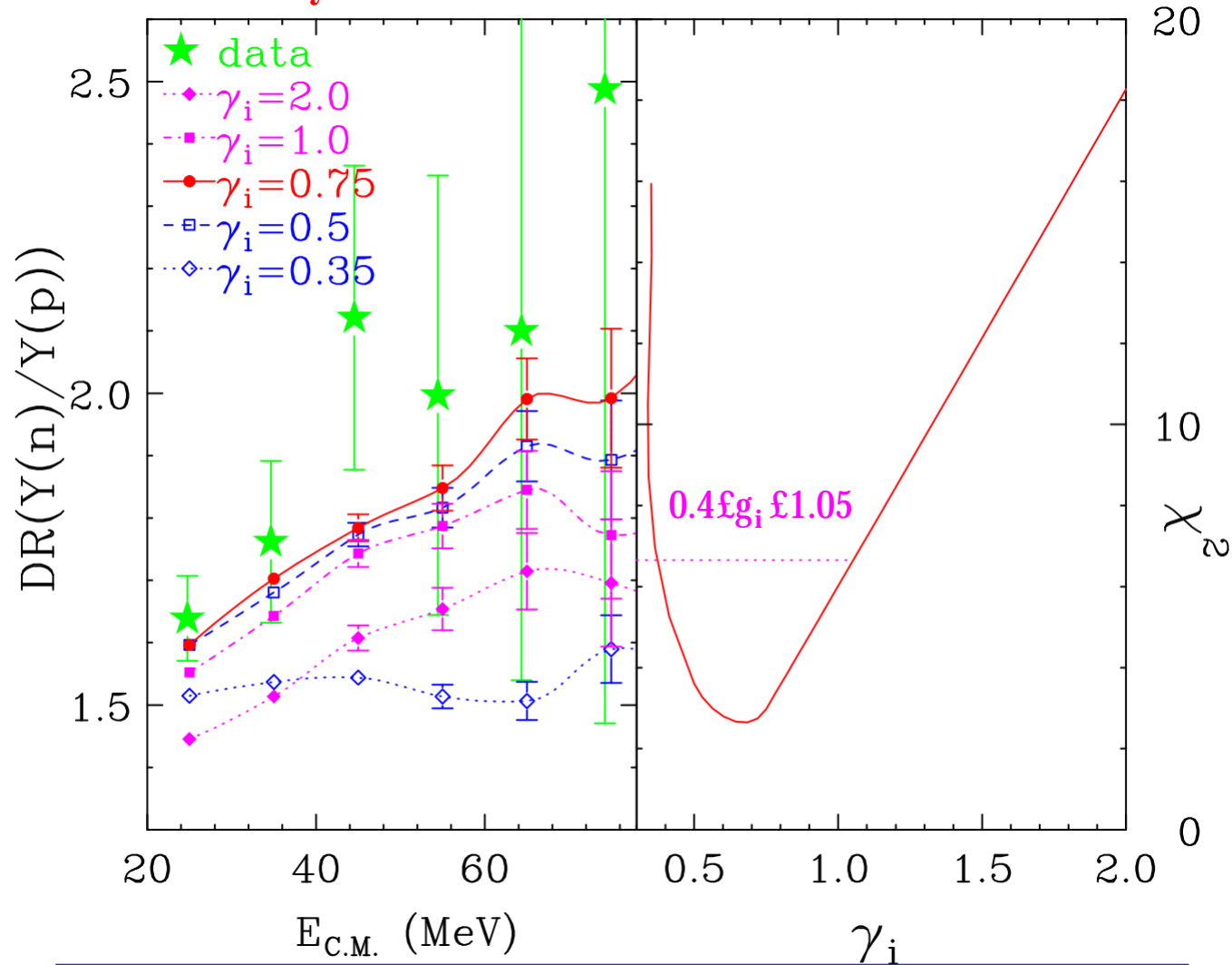


Data : Famiano et al. PRL 97 (2006) 052701

Cluster emission effects

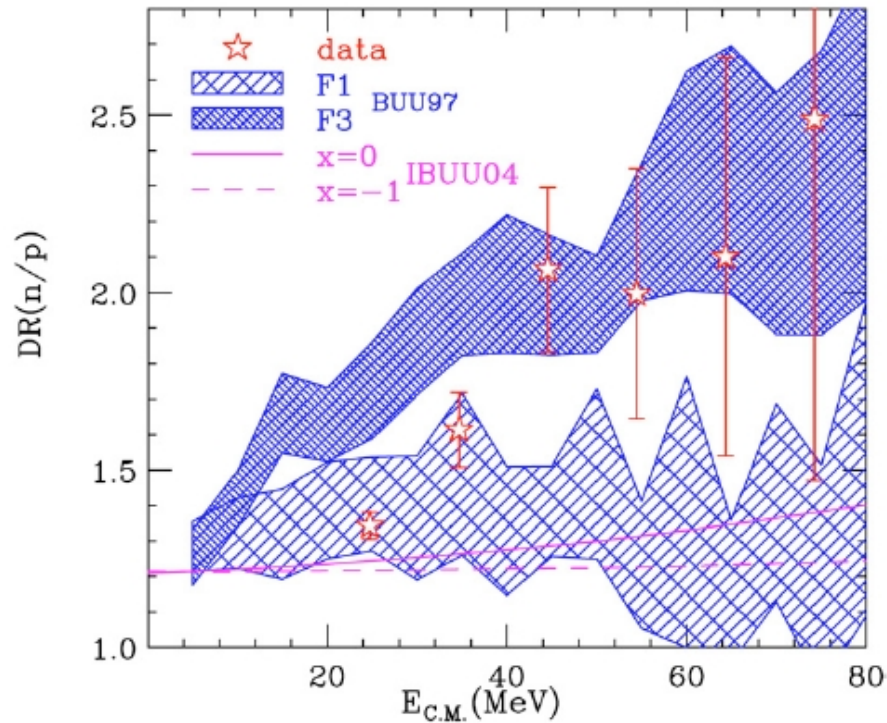
# Analysis of n/p ratios with ImQMD model

$$E_{\text{sym}} = 12.5(r/r_0)^{2/3} + 17.6(r/r_0)^{g_i}$$

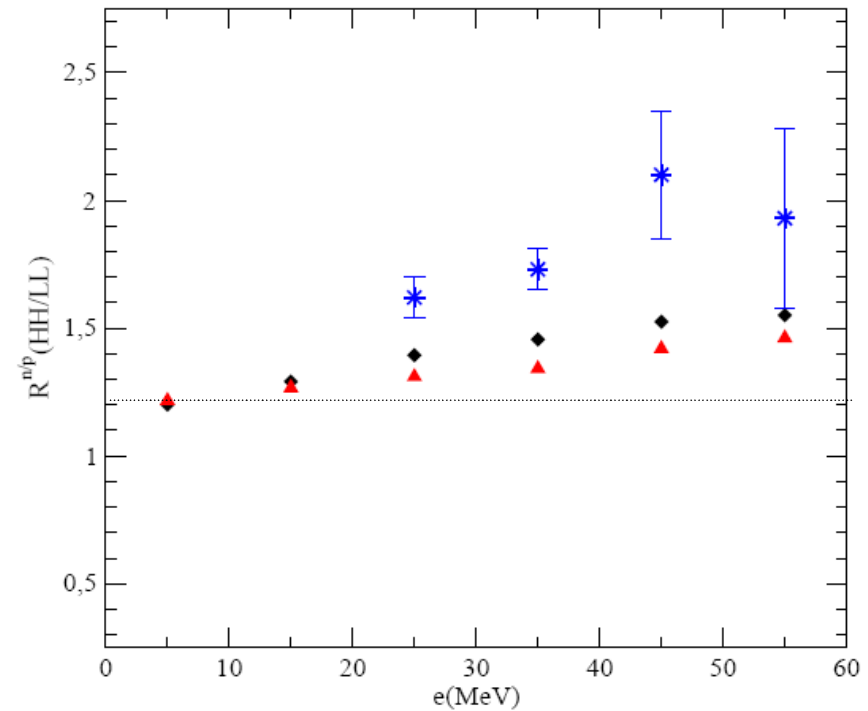


*Data need better measurements but the trends and magnitudes still give meaningful  $\chi^2$  analysis at  $2\sigma$  level*

## Ø Compare with other models



BUU results, B.A.Li, PRL78,1644; PLB634,378



Wolter, et al., arXiv.0712.2187

What can we learn from these differences?

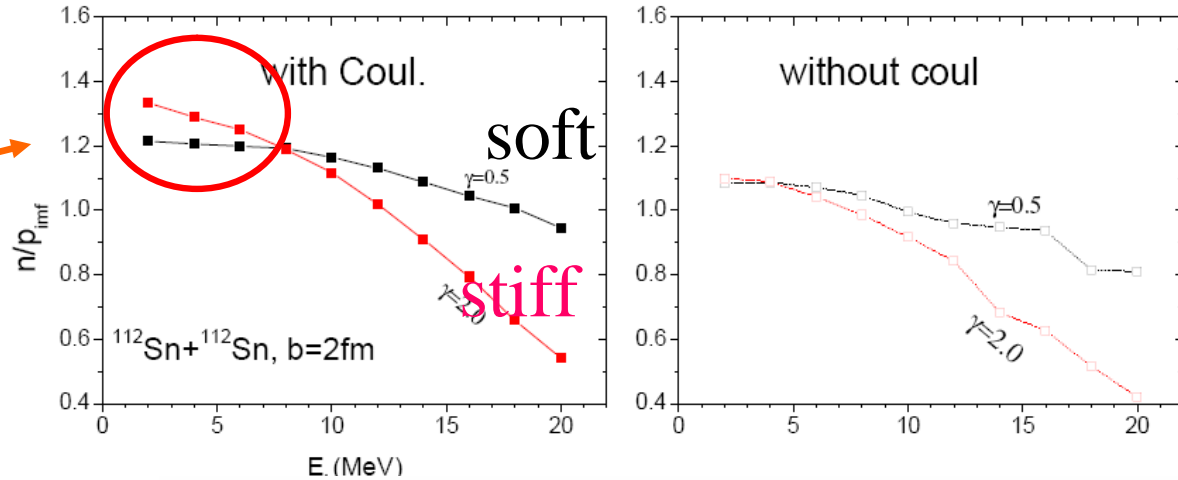
- *Is  $DR(n/p)$  sensitive to Isospin dependence of symmetry potential ?*
- *surface and surface asymmetry term*
- cluster formation mechanism
- others ?



### 3), N/Z for IMFs

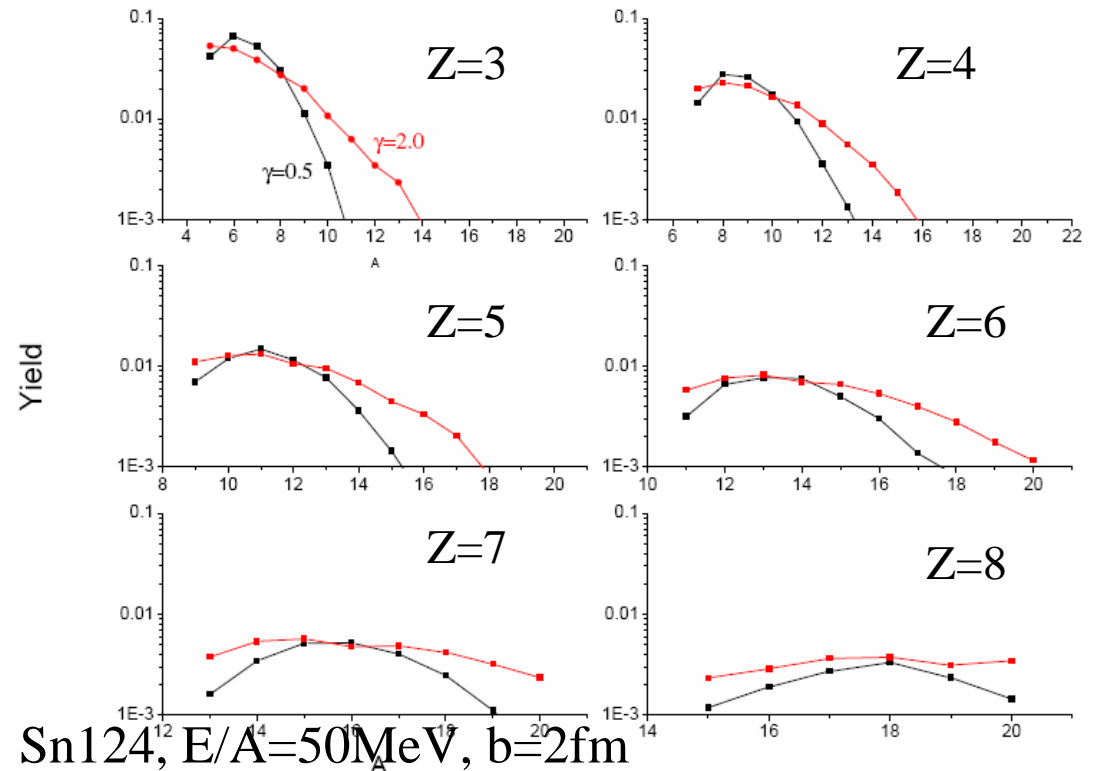
$$N/Z = \frac{\sum_i N_i}{\sum_i Z_i} \quad \text{for IMFs is sensitive to the } E_{\text{sym}}(\rho)$$

*Coul. effects*

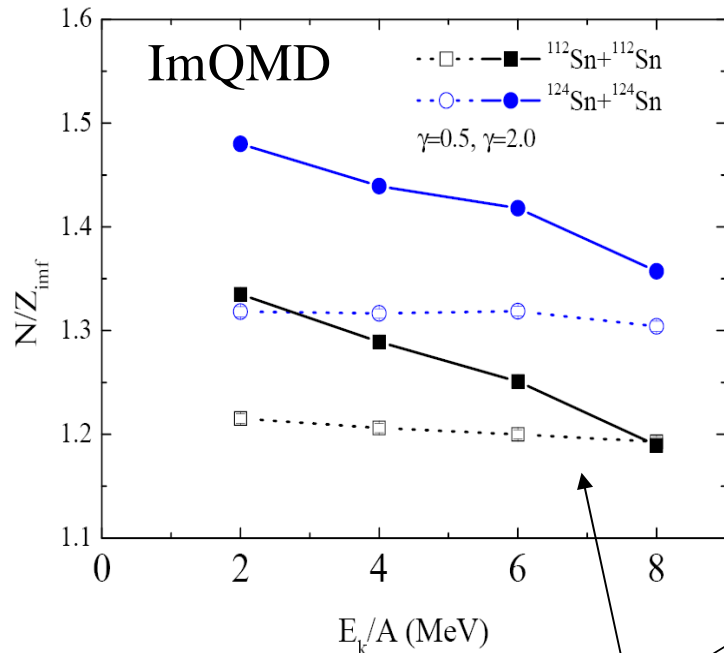


*Isostop distributions*

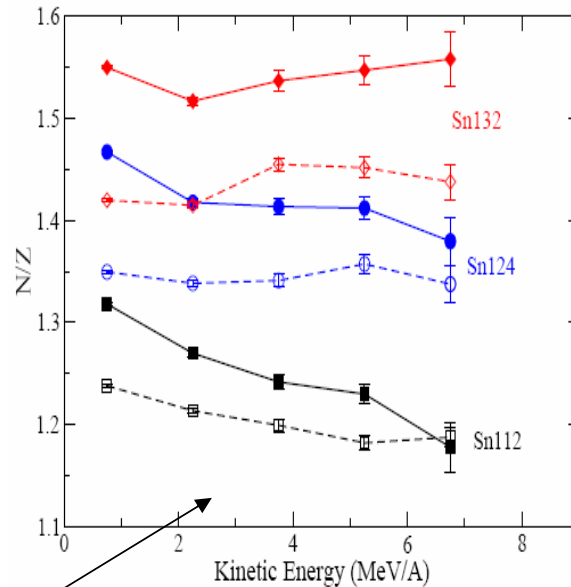
*For light particles, stiff symmetry potential make more neutron rich isotope*



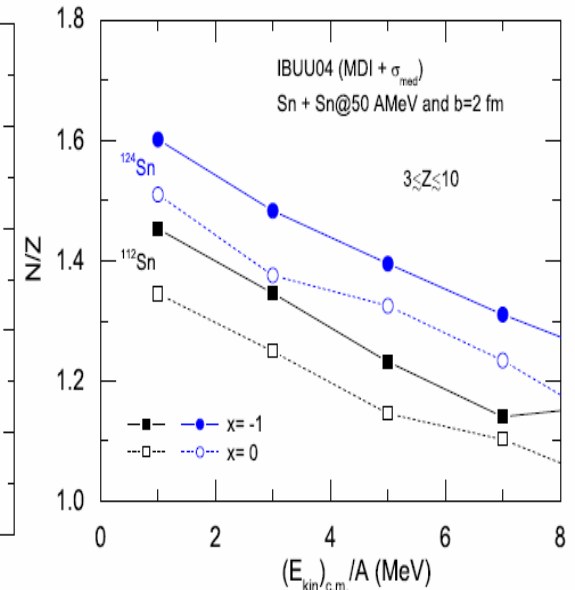
# Ø Compare with other models



M.Colonna, et al., arXiv 0707.3092



B.A.Li, PR464,113



IBUU04

Almost same

*Momentum independent  
 symmetry potential*

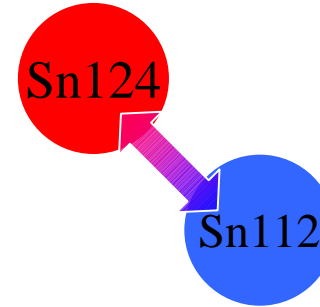
*Momentum  
 dependent symmetry  
 potential*

## 4), isospin diffusion- *Isospin transport ratio*

Isospin diffusion occurs only in asymmetric systems A+B

$$R_i = 2 \frac{x_{AB} - (x_{AA} + x_{BB}) / 2}{x_{AA} - x_{BB}}$$

Rami et al., PRL, 84, 1120 (2000)



$x_{AB}, y_{AB}$  experimental or theoretical observable for AB

$$y_{AB} = a x_{AB} + b$$

$$R_i(x_{AB}) = R_i(y_{AB})$$

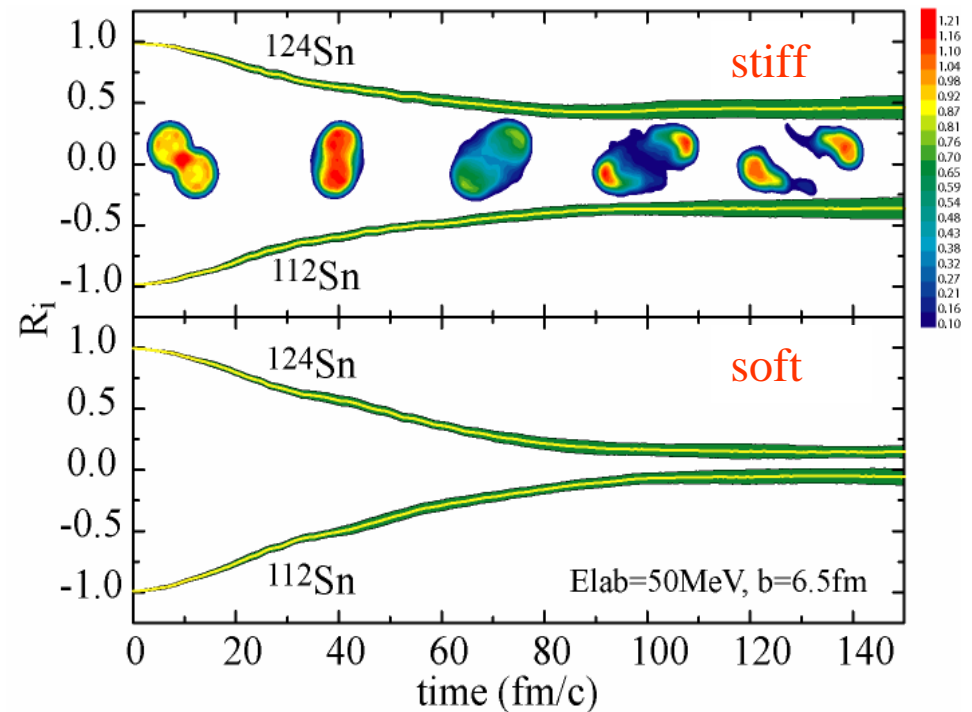
Ø Isospin “diffuse” through low-density neck region

Ø Symmetry energy drives system towards equilibrium.

*stiff EOS* → small diffusion;  $|R_i| \gg 0$

*soft EOS* → fast equilibrium;  $R_i \rightarrow 0$

Tsang, et al., PRL92, 062701(2004)



# Analysis of isospin diffusion data with ImQMD model

$$R_i = 2 \frac{x_{AB} - (x_{AA} + x_{BB})/2}{x_{AA} - x_{BB}}$$

$$S(\rho) = 12.5(r/r_0)^{2/3} + 17.6(r/r_0)^{g_i}$$

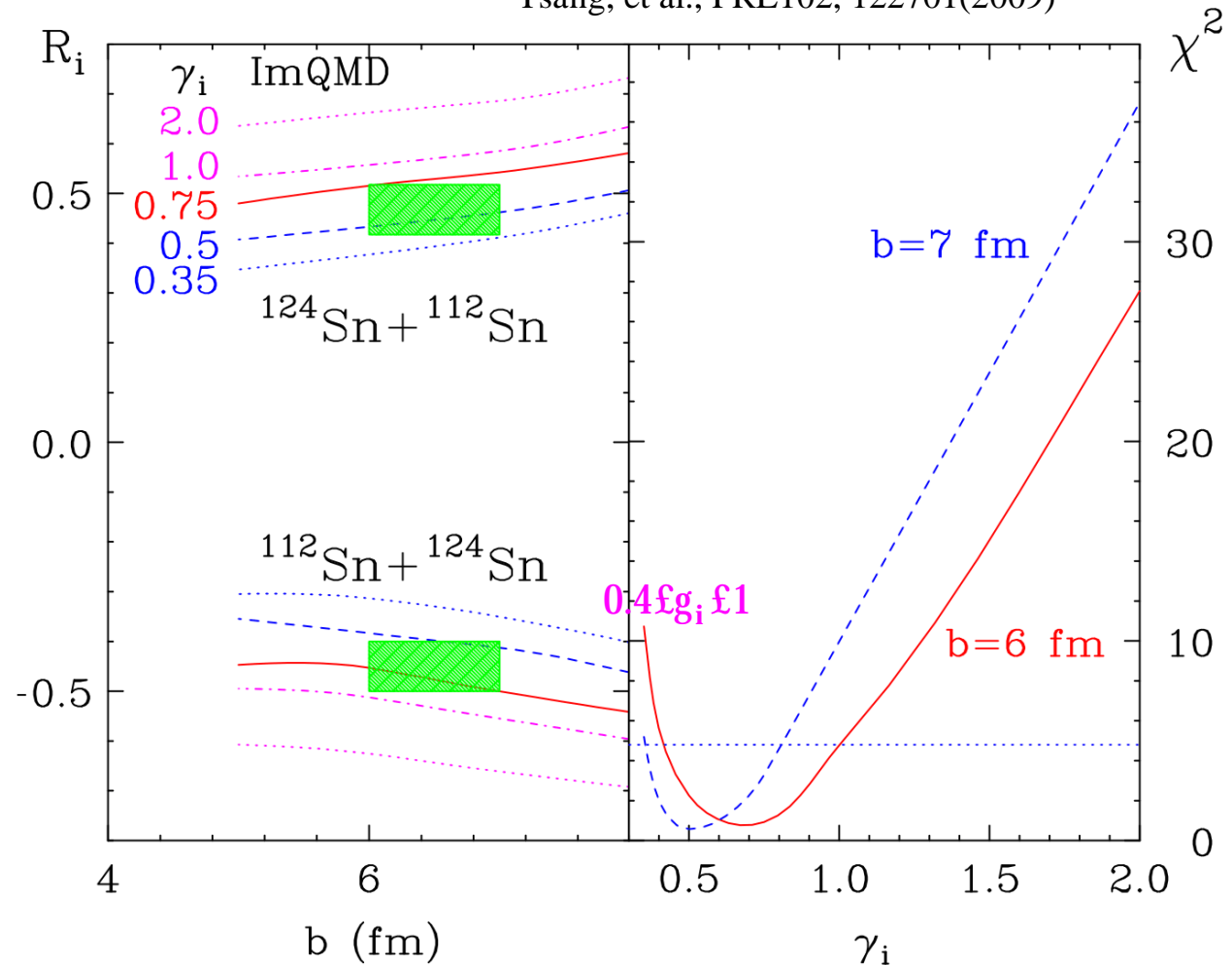
Tsang, et al., PRL102, 122701(2009)

$x(\text{data}) = \alpha$   
 $x(\text{QMD}) = \delta$

$\delta$  isospin  
 asymmetry of  
 emitting  
 source

Equilibrium  
 $R_i = 0$

No diffusion  
 $R_i = 1; R_i = -1$



# Analysis of rapidity dependence of $R_i$ with ImQMD model

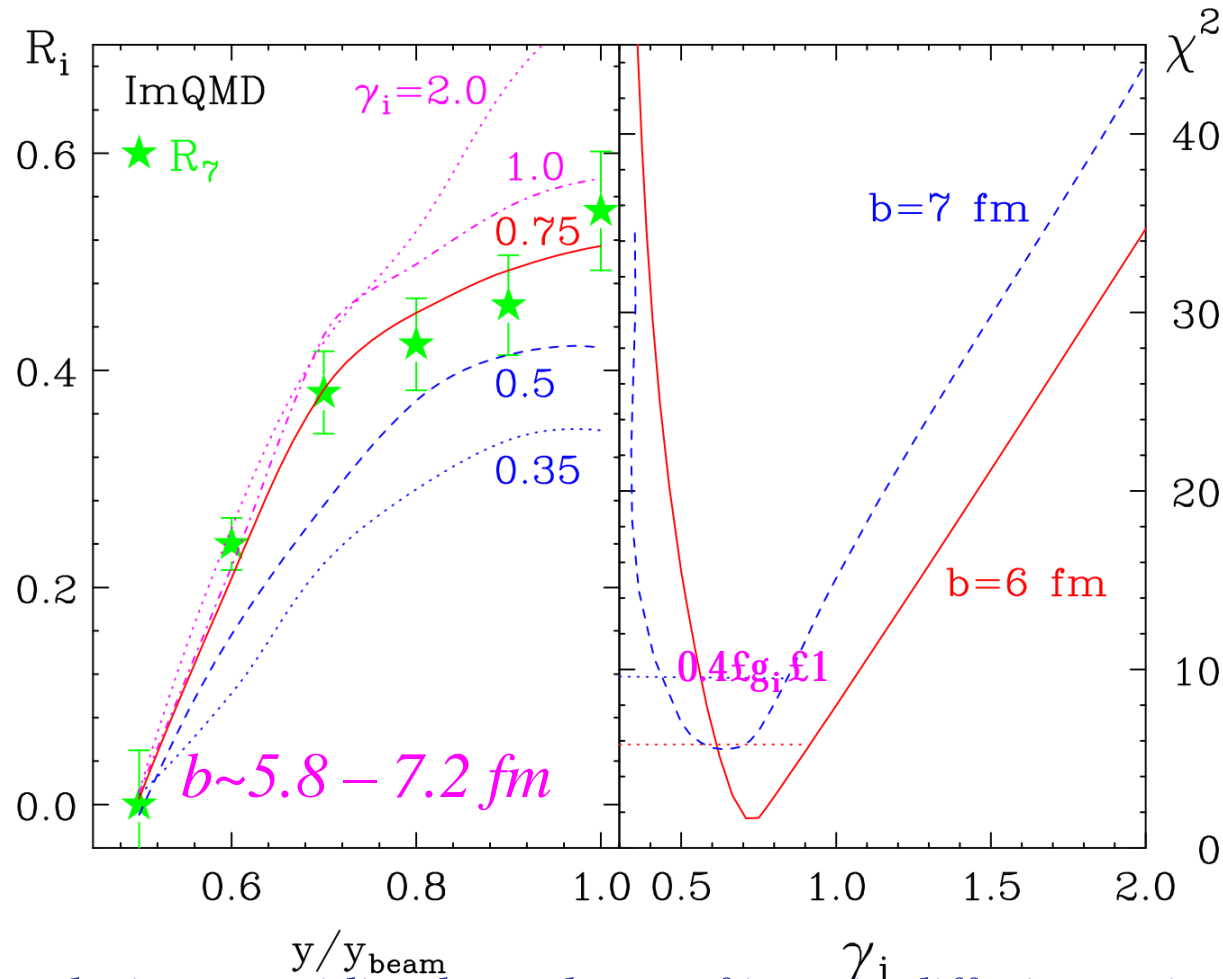
$$R_i = 2 \frac{x_{AB} - (x_{AA} + x_{BB})/2}{x_{AA} - x_{BB}}$$

$$S = 12.5(r/r_0)^{2/3} + 17.6(r/r_0)^{g_i}$$

$x(\text{data})$   
 $= Y(^7\text{Li}/^7\text{Be})$   
 $x(\text{QMD}) = \delta$

Equilibrium  
 $R_i = 0$

No diffusion  
 $R_i = 1; R_i = -1$

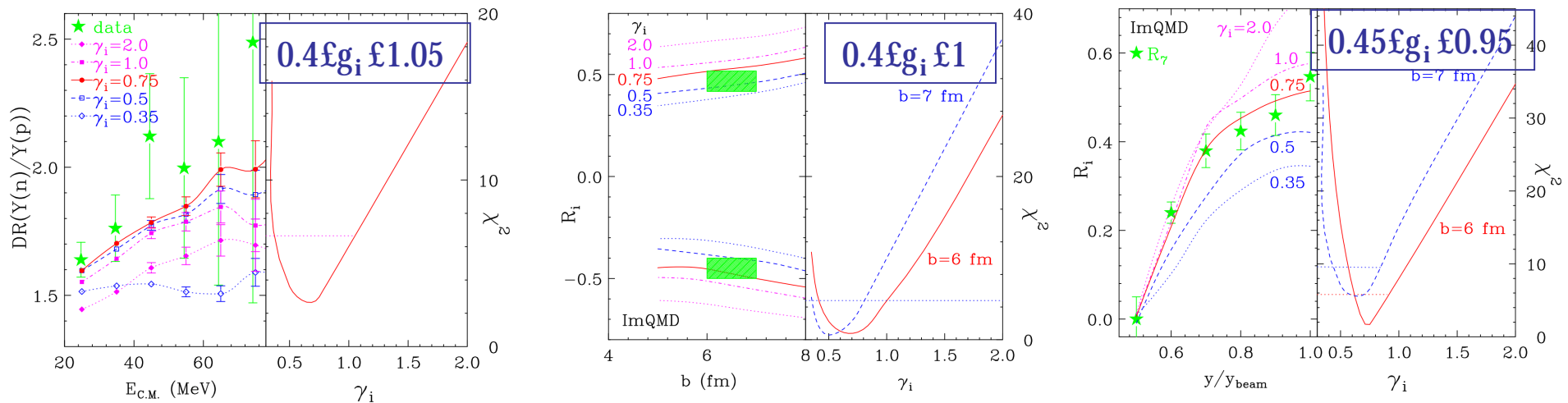


*New analysis on rapidity dependence of isospin diffusion ratios – not possible with BUU type of simulations due to lack of fragments.*

- Consistent constraints from the  $\chi^2$  analysis of three observables from ImQMD model

$$S(r) = 12.5(r/r_0)^{2/3} + 17.6(r/r_0)^{g_i} \quad 0.4 \leq g_i \leq 1, \text{ best fit } g_i \sim 0.7$$

ImQMD describes np ratios and two isospin diffusion measurements



Consistent constraints from the  $\chi^2$  analysis of three observables

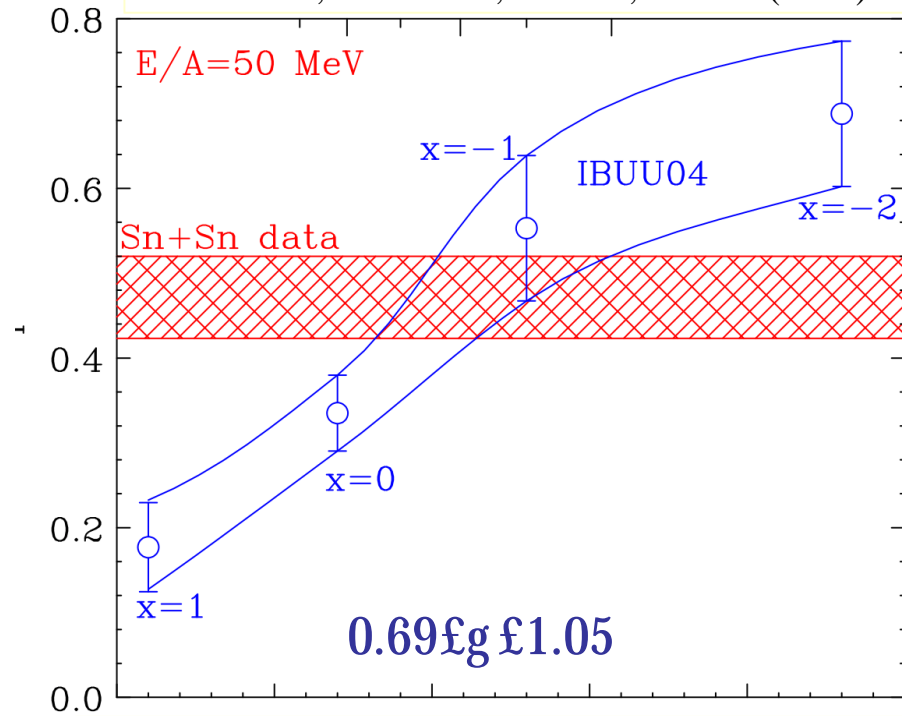
$$S(\rho) = 12.5(r/r_0)^{2/3} + 17.6(r/r_0)^{g_i}$$

$$0.4 \leq g_i \leq 1$$

# Ø Compare with other transport models

**IBUU04 :  $S(\rho) \sim 31.6(r/r_0)^g$**

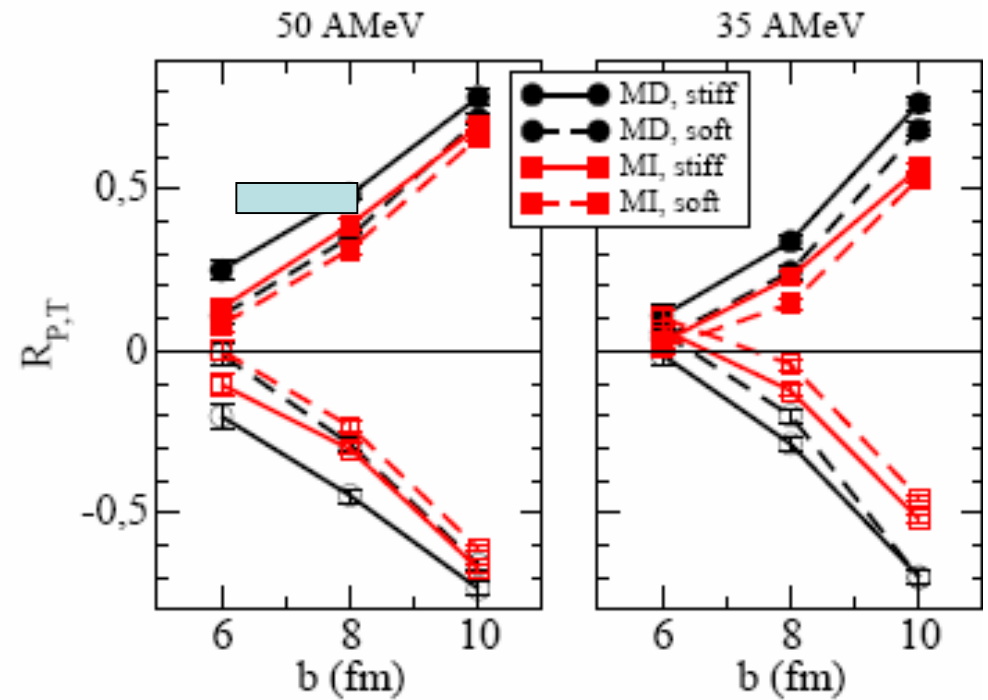
L.W. Chen, ... B.A. Li, PRL 94, 032701 (2005)



stiffness  $\longrightarrow$

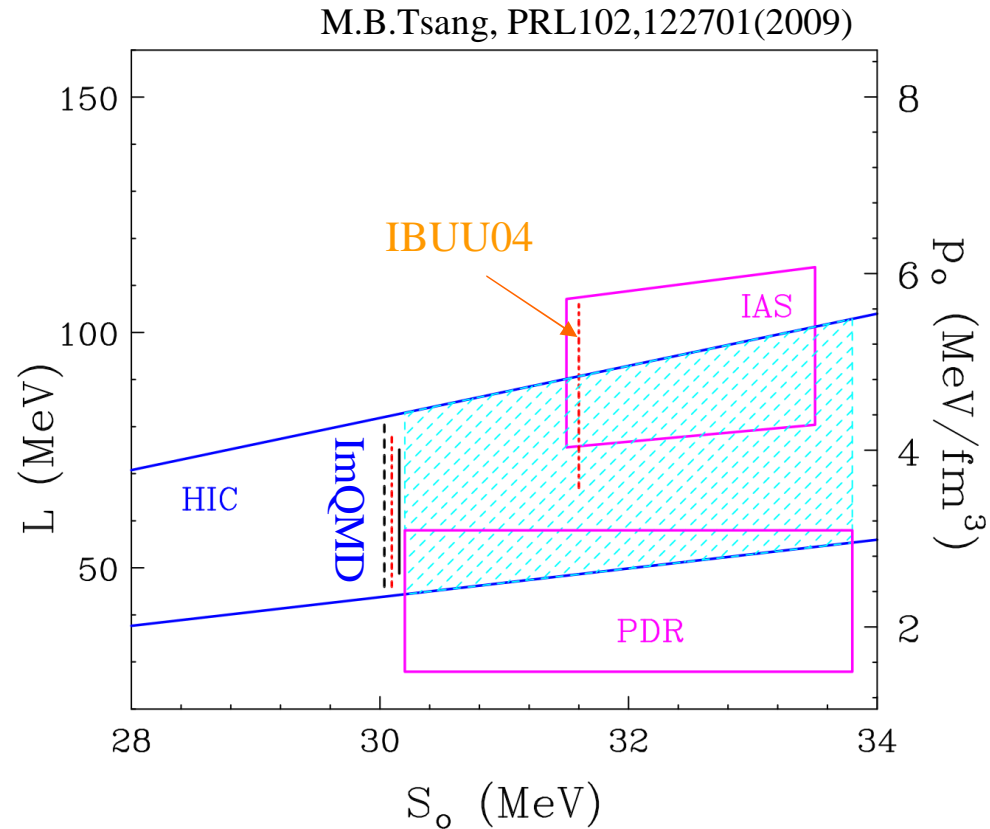
## Isoscalar MDI

SMF, Wotler, arXiv:0712.2187



Stiff:  $\gamma \sim 1.0$   
Soft: SkM\*

# Constraints from different model



Ø Results from ImQMD overlap with IBUU04 constraints and others.

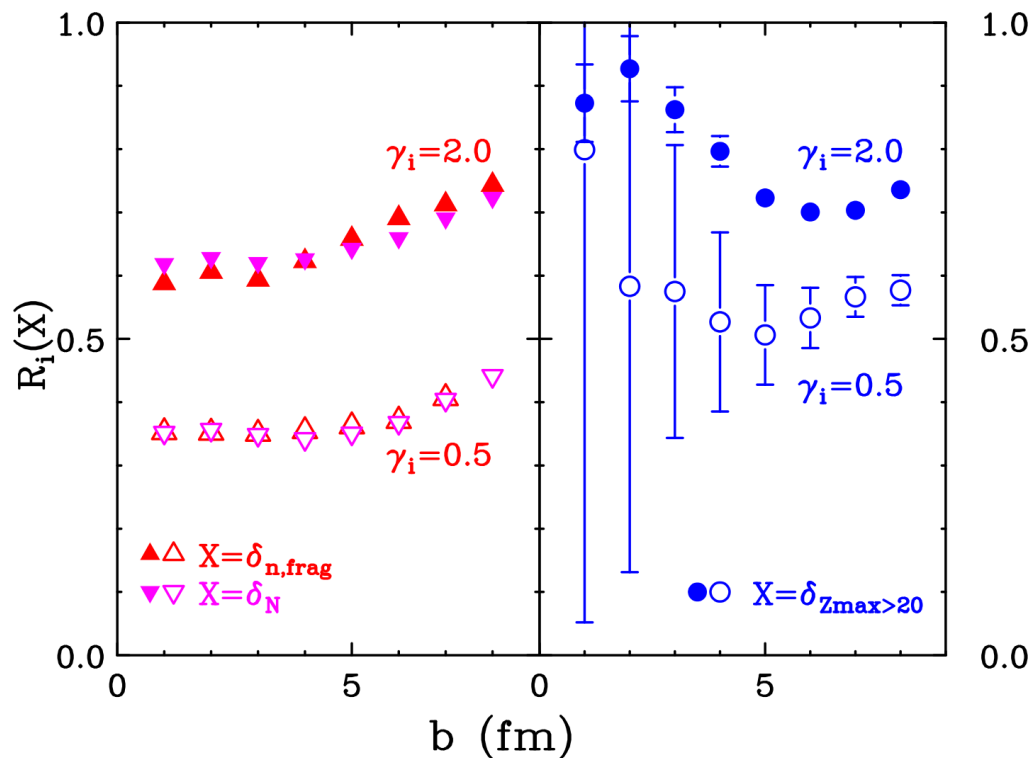
Ø *Is  $R_i$  not sensitive to the momentum dependent symmetry potential?*



# ∅ Ri values depend on the tracer selection

Three definitions on the isospin asymmetry  $X=\delta$  :

1. N & Frags (with v cuts)
2. N (with v cuts)
3.  $Z_{max}>20$



• *Ri values depend on the tracer*

•  $R(X=d(Z_{max}>20)) > R(X=d(N\&Frag))$

## 3, Summary

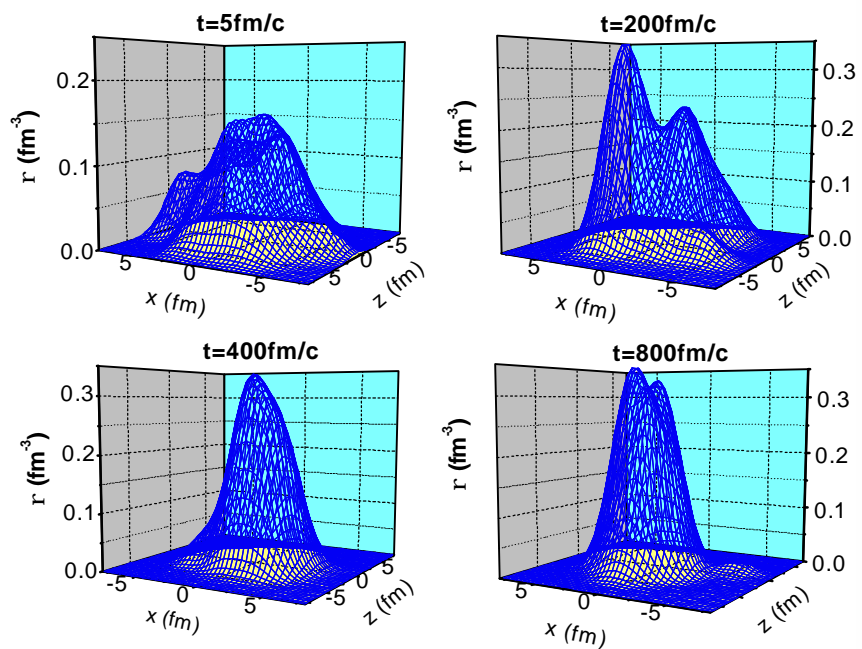
1. Both  $DR(n/p)$  and  $R_i$  have been consistently described by ImQMD model. The consistency between the different probes of symmetry energy suggests that increasing stringent constraints on the symmetry energy at subsaturation density can be expected.
2. The values of isospin transport ratio  $R_i$  sensitive to the symmetry potential and its also depend on the tracer.
3. Momentum dependent symmetry potential, effective mass splitting and in medium cross section should be further explored.
4. Theoretical uncertainty on the transport model need to be estimated.

# Acknowledgements

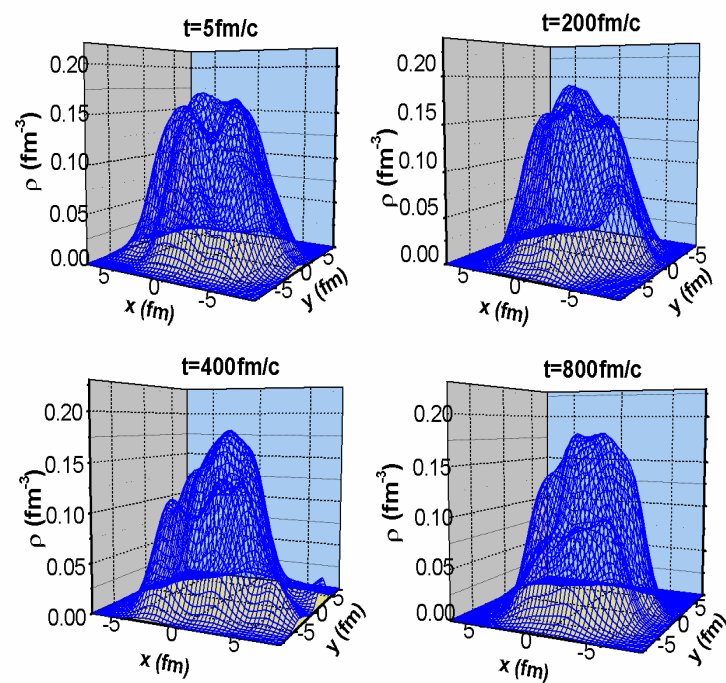
M.B.Tsang, Zhuxia Li, P. Danielewicz, W.G. Lynch,

# Surface term contributions

$^{90}\text{Zr}$  without surface term



$^{90}\text{Zr}$  with surface term

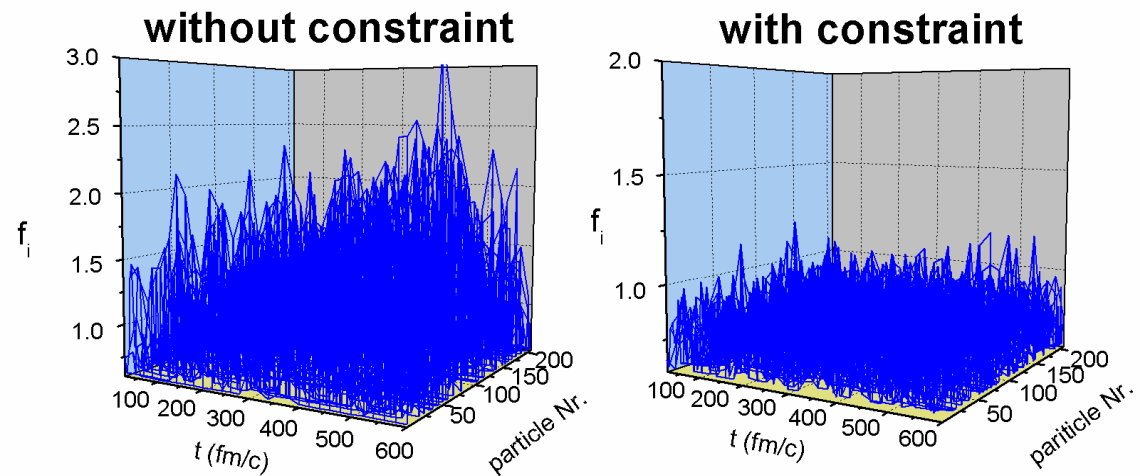




# Effect of phase space constraint (approx. treatment of the anti-symmetrization)

<sup>208</sup>Pb phase space dis.

$$\bar{f}_i \leq 1$$



(The details of the treatment, see CoMD, Phys. Rev. C 64, 024612).